Gravitational Collapse With Distributed Adaptive Mesh Refinement

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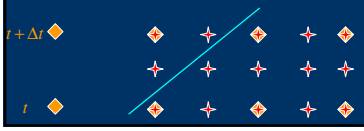
Progress on different fronts:

- Novel AMR Boundary Treatment
- Gravity
- Magnetohydrodynamics (MHD)
- Cell Centered/Flux Conservative Fluid
- Nonlinear Wavemaps

Tapered Boundary

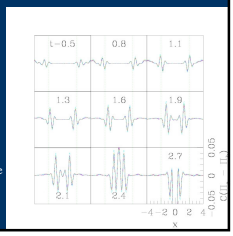
L.Lehner, SLL, O.Reula: gr-qc/0510111

- Fine grids have artificial boundaries w/in computational domain
- Require information from parent/coarse grid
- Discard points causally connected to boundary



Tapered Boundary

- No interpolation in time
- Easily extends to higher order
- Example: 3rd order derivatives w/ a nonlinear scalar field demonstrates:
  - Excellent convergence
  - Good "transmission"



Full GR in 3D

- 1st Order symmetric hyperbolic scheme of
  M. Tiglio, L.Lehner, D.Neilsen: gr-qc/0312001
- Derivative operators satisfying *summation by parts*
- Maximally dissipative outer boundary conditions…set time derivative of incoming modes to zero
- Time harmonic lapse

Gravitational Collapse of Brill

- Initial data generated:
  - Axisymmetric Vacuum solutions

$$ds^2 = \Psi^4\left(e^{2q}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right)$$

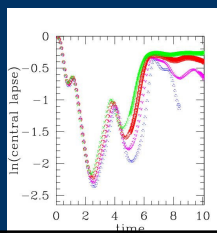
  - Where $q = \dfrac{A\rho^2}{1+r^{10}}$ (Eppley's form)
- Choose amplitude A, determine if black hole forms:
  - Apparent horizon finder
  - Collapse of lapse

Gravitational Collapse of Brill

- Look for threshold of black hole formation:
  - A>A* – black hole forms
  - A=A* – critical point
  - A<A* – energy disperses
- Difficulties:
  - Horizon finder issues
  - In contrast to maximal slicing, lapse may not "know" global property of hole formation

Preliminary Indications

- Evolutions for different amplitudes A
- A=2.24, 2.27, 2.30, 2.33



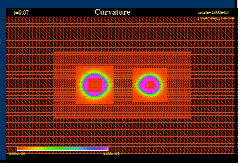
Black Hole Excision

- Excise black hole singularity from computation
- Choose excision region small enough to ensure all modes incoming
- Excise along cubes to simplify derivative operators which satisfy SBP
- Modified Kreiss-Oliger dissipation operator
- See: gr-qc/0312001

Moving Excision

- Assume holes contained within finest level
- Repopulate via linear extrapolation
- Repopulate only 1 grid point width per time step

- Single black hole evolves a long time
- Two black holes still needs work (dynamic horizon finder)



Magnetohydrodynamics (MHD)

- Vertex Centered, HRSC code

(see talks by Anderson, Neilsen, Hirschmann):
D.Neilsen, E.W.Hirschmann, R.S.Millward, gr-qc/0512147

- Need to keep divergence constraint satisfied
- One option: elliptic divergence cleaning

1. Solve elliptic equation via Multigrid method:

$$\nabla^2 \psi = \vec{\nabla} \bullet \vec{B}$$

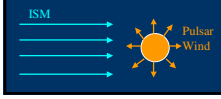
2. Correct magnetic field:

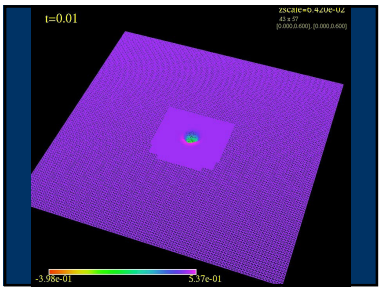
$$\vec{B} \longrightarrow \vec{B} - \vec{\nabla}\psi$$

Application of MHD: Pulsar Wind Nebulae (PWN)

W/ Tanvir Rahman (L7.00007 4:27pm)

- Model the interaction between:
  - A pulsar's "spin down luminosity"
  - ISM in which pulsar is moving
- Look for bow shock structure…match against observations





Cell Centered Evolutions

W/ Patrick Motl, Tanvir Rahman

- Fluid codes generally use integral form of equations
- Flux conservative schemes explicitly conserve the evolved quantities
- Very useful for certain situations:
  - Stars—conserve mass
  - Binaries—conserve angular momentum
- No conservative scheme w/ vertex centered AMR

S. Li, J.M. Hyman: LA-UR-03-8927

- We can define fields as **either** vertex **or** cell centered
- Still need to implement coarse grid correction

Nonlinear Wavemaps

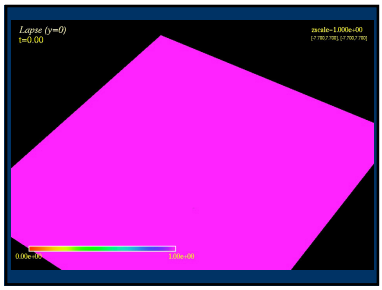
W/ Jason Williams

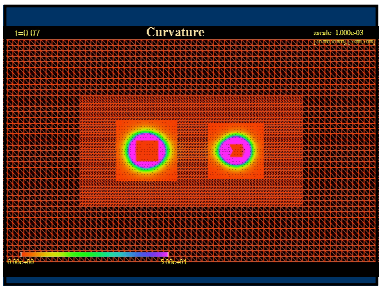
- Flatspace w/ scalar fields
- Scalar fields act as coordinates-on/map-to a target space
- So-called *Harmonic Map:*
  - From base space: Minkowski 3+1
  - To target space: constant curvature 2D surface (S^2 or hyperboloid)
- Interesting dynamics, expect critical behavior at threshold of singularity formation

Summary

- Modeling full GR w/ distributed AMR:
  - Black hole critical behavior
  - Dynamic black hole excision
- MHD and GRMHD promise lots of physics
- Study coupling of cell-/vertex centered fields
- More interesting wave map results

NSF Support PHY-0325224


Lapse (y=0)
t=0.00


Curvature